Direct Research Journal of Engineering and Information Technology (DRJEIT) Vol.2 (2), pp. 17-24, May 2014
ISSN 2354-4155 Available online at directresearchpublisher.org/drjet

The dynamics of the wagon rolling down the hump profile under the impact of fair wind

Research Paper

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Accepted 12 April 2014

The article gives an account of the results of the construction of mathematical model of the wagon rolling down the hump profile under the impact of the fair wind. There have been derived analytical formulas for determining wagon rolling speed and distance travelled down the first profile hump section. The results of the research obtained can be used for all the rest hump sections with the allowance made for the specifics of braking forces on these sections.

Key words: Speed and direction of fair wind, D'Alembert principle, rational function integral and integral containing trigonometric function, speed of wagon rolling down the first profile hump section, distance travelled.

INTRODUCTION

The statement of the problem and its connection to research and practical tasks

In Khabibulla, (2014) contains a detailed account of the results of analytical investigation concerning the determination of the speed and the distance travelled by the wagon rolling down hump profile under the fair wind.

However, up to now scant notice has been attracted to the dynamics of wagon rolling down the hump profile under the fair wind with strict adherence to classical statements of theoretical mechanics. For this reason, the construction of mathematical model of the wagon rolling down the hump profile under the impact of the fair wind still remains a very urgent issue.

The aim is to mathematically describe the dynamics of the wagon rolling down the hump profile under the impact of fair wind.

In the long term the research results obtained can be used in solving a technical problem of determining hump rational geometrical parameters and kinematic characteristics of the wagon rolling down a hump.

Formulation of a problem

It is necessary to determine the speed and the distance traveled by the wagon rolling down the hump profile under the impact of gravity and fair wind.

SOLUTION METHODS

We will make use of the following classic statements of theoretical mechanics: the theorem of velocity additions in compound movement, the principle of release from constraints, the fundamental law of dynamics Loitsyansky, (1983) and the major notions of differential and integral calculus (Bronstein, 1980, Piskunov,1978).

Problem specification and agreed preconditions

Just as in (Khabibulla, 2014 and Komarov, 2004) a general case when a wagon is rolling progressively down the hump at specified initial speed v_0 (normally $4 \div 5$ km/h or $1{,}1 \div 1{,}38$ m/s).When a single wagon (or a cut) is rolling down the hump the wagon will mainly experience the impact of external forces in the form of gravity forces of the wagon with cargo or without it – $\overline{G}$ and aerodynamic resistance force $\overline{F}_w$ where

$$(\overline{F}'_{wx}, \overline{F}'_{wy}) \in \overline{F}_w) \qquad (1)$$

In (Khabibulla, 2014 and Komarov, 2004) let the wagon move progressively at transferring speed

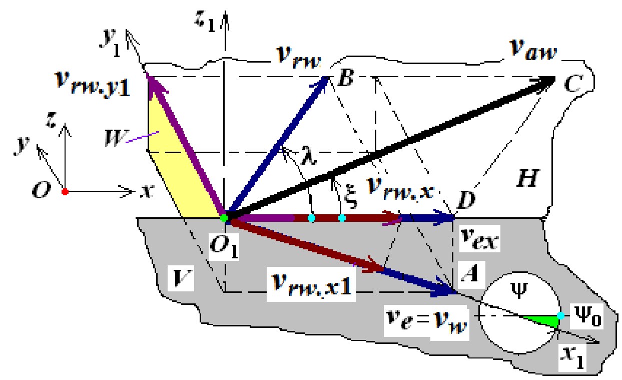

Figure 1. Vectorial diagram of wagon and fair wind speed.

$\overline{v}_e = \overline{v} = \overline{v}_w$ (*the value to be found*) down the hump in respect to immobile coordinate system *Oxyz* (Figure 1). Wind speed in respect to the hump crest (ground) (that is the absolute speed of air particles) $\overline{v}_{a.w}$ (*the value being set*) is directed across along axis *Oxyz*. We assume that mobile coordinate system $O_1x_1y_1z_1$, is rigidly bound to the wagon while air particles in their turn move at speed $\overline{v}_{rw}$ in respect to mobile coordinate system $O_1x_1y_1z_1$ (that is the wagon). It is necessary to find projections of air particle relative speed (wind speed) (in respect to mobile axes $O_1x_1y_1z_1$ bound to the wagon).Figure 1 has the following table of symbols: *O* is the beginning of immobile coordinate system *Oxyz*, rigidly bound to the hump crest, O_1 is the beginning of mobile coordinate system $O_1x_1y_1z_1$, rigidly bound to the wagon, *H*, *V* and *W* are horizontal, vertical and frontal planes; ψ_0 is a descend angle (in accordance with hump profile (*the value to be set*); $\overline{v}_{rw}$ is relative speed of air particles (wind speed) in respect to mobile reference frame$O_1x_1y_1z_1$ (wagon) (*the value to be calculated*); λ is a direction angle of the vector of relative air particle speed along axis *Ox* (*the value to be calculated*); $\overline{v}_{a.w}$ is air particle speed in respect to the ground (to the hump crest) (according to the data of Construction Norms and Regulations this is *the value to be set*); ξ is a direction angle of vector of air particle absolute speed along axis *Ox* (*the value to be set*).

Relative air particle speed (wind speed) $\overline{v}_{r.w}$ was consider and was located on horizontal plane *H* and directed at angle λ (or λ_0) relative to horizon (axis *Ox*), while transferring speed (wagon speed) $\overline{v}_e = \overline{v} = \overline{v}_w$ is located on vertical plane *V* and is directed at descend angle ψ (or ψ_0) relative to horizon (*Ox*).

Let us show the dependences of projections of air particle relative speed (wind speed) $\overline{v}_{r.w}$ (*the value to be calculated*) from wind speed in relation to the hump crest (ground) (that is air particle absolute speed) $\overline{v}_{a.w}$ (*the value to be set*) and wagon speed $\overline{v} = \overline{v}_w$ (*the value to be found*). In accordance with the theorem of velocity addition (Loitsyansky, 1983 and Komarov, 2004) we will write:

$$\overline{v}_{a.w} = \overline{v}_{ex} + \overline{v}_{r.w} \tag{1}$$

where $\overline{v}_{a.w}$ is air particle absolute speed (wind speed); $\overline{v}_{ex} = \overline{v}_x = \overline{v}_{wx}$ is the projection of transferring speed (wagon speed) $\overline{v}_e = \overline{v} = \overline{v}_w$ onto axis *Ox*:

$$v_{ex} = v_x = v_{wx} = v_e \cos(\psi_0) \tag{2}$$

with allowance made for the fact that ψ (or ψ_0) is a descend angle of the hump to the horizon (axis *Ox*);

$\overline{v}_{r.w}$ is air particle relative speed (wind speed) in relation to the wagon.

As we consider the direction of the wind to coincide with the direction of the wagon (i. e. with the fair wind) the projection (1) onto axis *Ox* has the form of:

$$v_{a.w} \cos(\xi) = v_e \cos(\psi_0) + v_{rw.x}$$

Hence

$$v_{rw.x} = v_{a.w} \cos(\xi) - v_e \cos(\psi_0) \tag{3}$$

where is an angle between resultant vector $\overline{v}_{a.w}$ (air particle absolute speed (wind speed) and longitudinal axis *Ox*, rad.

In the expression the module of air particle relative speed (that is wind speed in relation to the wagon) $\overline{v}_{r.B}$ is to be found according to the cosine theorem (Bronstein, 1980 and Vodnev,1998):

$$v_{r.w} = \sqrt{v_{ex}^2 + v_{a.w}^2 - 2v_{ex}v_{a.w}\cos(\xi)}$$

Direction angle λ of air particle relative speed (wind

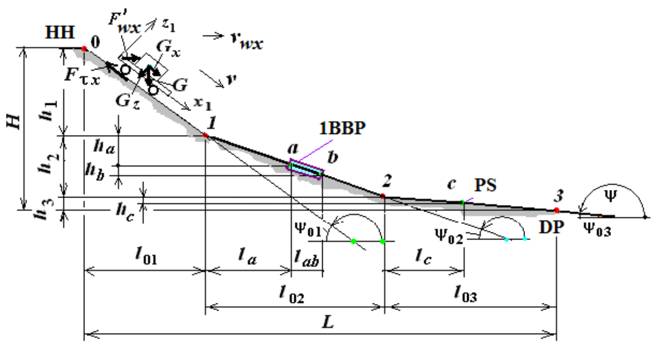

Figure 2. Profile of various hump sections.

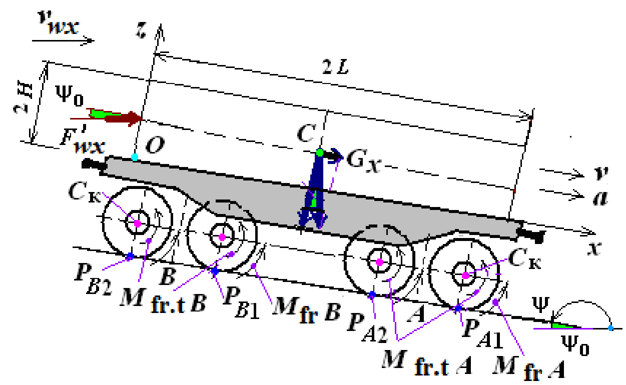

Figure 3. Simplified model of wagon rolling down the hump.

speed) $\overline{v}_{r.w}$ as in (Vodnev,1998) is found according to the theorem of sines:

$$\sin(\lambda) = \frac{v_{a.w}}{v_{r.w}} \sin(\xi).$$

In accordance with (Bronstein,1980) aerodynamic resistance force $\overline{F}_{rw}$ for fair wind on axis *Ox* and on axis *Oy*, unlike in (Khabibulla, 2014) is found, N:

$$F'_{rwx} = 0.5 c_{\mathrm{a}} \rho_{\mathrm{a}} A_{\mathrm{EV}} \left(v_{a.w} \cos(\xi) - v_e \cos(\psi_0)\right)^2 \tag{4}$$

$$F'_{rw.y} = 0{,}5 c_a \rho_{\mathrm{a}} A_{\mathrm{TV}} \left(v_{r.w} \sin(\lambda)\right)^2 \tag{5}$$

In (Piskunov, 1978 and Komarov,2004) there is the following symbol table: c_a is infinite coefficient of air resistance, dependent on the form of the solid and the way it is oriented in the process of motion (normally it is accepted according to the form of the solid surface to vary from 0.55 to 1.2, for example, cylindrical solids with a circle as cross section (pipe) c_a = 0.6, for flat surfaces c_a: c_a = 1.1); ρ_a is average air density (kg/m^3) (normally accepted to be 1.26 ÷ 1.29); A_{EV} is the area of the end surface of the wagon with cargo, m^2; $A_{EV} = 2B{\times}2H$ (where 2*B* и 2*H* are width and height of windward surfaces of the wagon with cargo, m); A_{TV} is the area of end surfaces of the wagon with cargo: $A_{TV} = 2L \times 2H$ (where 2*L* is the length of end windward surfaces of the wagon with cargo, m), m^2.

Let us assume that a hump as in (Khabibulla,2014) consists of three sections 0 – 1, 1 – 1, and 2 – 3 linked by two breaking points 1 and 2. Let the first braking position (1st BP) with coordinates *a* and *b* be located on the second section 1 – 2, while on section 2 3 there is a point switch (PS) with coordinate *c* (Figure 2). Figure 2 has the following table of symbols: HH is hump height ; *H* and *L* is design height of the hump, m; h_1, h_2, h_3 and l_{01}, l_{02}, l_{03} are height and length of the corresponding hump sections; l_{ab} is length of the first braking position, m; ψ_{01}, ψ_{02} и ψ_{03} are slope angles of the corresponding hump sections, rad.; DT is design point.

Formation of rolling wagon design model

The problem was solved as a general case: the impacts of fair wind on the wagon (Figure 1), passing through rail joints, impact ascend up the point, ascend up the side track, change of air density etc. The model shown in (Figure 3) is taken as a simplified model of the wagon rolling down the hump with allowance made for rolling friction of wagon wheels with sliding, while the model presented in Figure 4 is taken as a design model (Khabibulla,2014).Figure 3 has the following table of symbols: $M_{\mathrm{frt}A}$ ($\{M_{\mathrm{frt}A1}, M_{\mathrm{frt}A2}, M_{\mathrm{frt}A'1}, M_{\mathrm{frt}A'2}\} \in M_{\mathrm{frt}A}$ and $M_{\mathrm{frt}B}$ ($\{M_{\mathrm{frt}B1}, M_{\mathrm{frt}B2}, M_{\mathrm{frt}B'1}, M_{\mathrm{frt}B'2}\} \in M_{\mathrm{frt}B}$ are internal forces in the form of rolling frictional torque in bearings of journal box units of the leading truck *A* and rear truck *B*, $M_{\mathrm{frt}} = M_{\mathrm{frt}A} + M_{\mathrm{frt}B}$; P_{A1}, P_{A2}, P_{B1}, P_{B2}, being speed momentary centers (Khabibulla,2014).

In Figure 4, allowance is made for the fact that $\{\overline{N}_{A1}, \overline{N}_{A2}\} \in \overline{N}_A$, $\{\overline{N}_{A'1}, \overline{N}_{A'2}\} \in \overline{N}_{A'}$, $\{\overline{N}_{B1}, \overline{N}_{B2}\} \in \overline{N}_B$, $\{\overline{N}_{B'1}, \overline{N}_{B'2}\} \in \overline{N}_{B'}$, $\{F'_{\tau A'}, F''_{\tau A'}\} \in \overline{F}_{\tau A'}$, $\{F'_{\tau B}, F''_{\tau B}\} \in \overline{F}_{\tau B}$, $\{F'_{\tau B'}, F''_{\tau B'}\} \in \overline{F}_{\tau B'}$ are normal and tangent components of rail thread reactions. In so doing allowance is made for the fact that $\overline{F}_{\tau A}$, $\overline{F}_{\tau A'}$, $\overline{F}_{\tau B}$, $\overline{F}_{\tau B'}$, $\overline{F}_{\tau B'W}$ are cohesion friction forces acting between contacting wheel surfaces and rail threads, that is $\overline{F}_{\tau A} = \overline{F}_{\mathrm{fr}A}$, $\overline{F}_{\tau A'} = \overline{F}_{\mathrm{fr}A'}$, $\overline{F}_{\tau B} = \overline{F}_{\mathrm{fr}B}$,

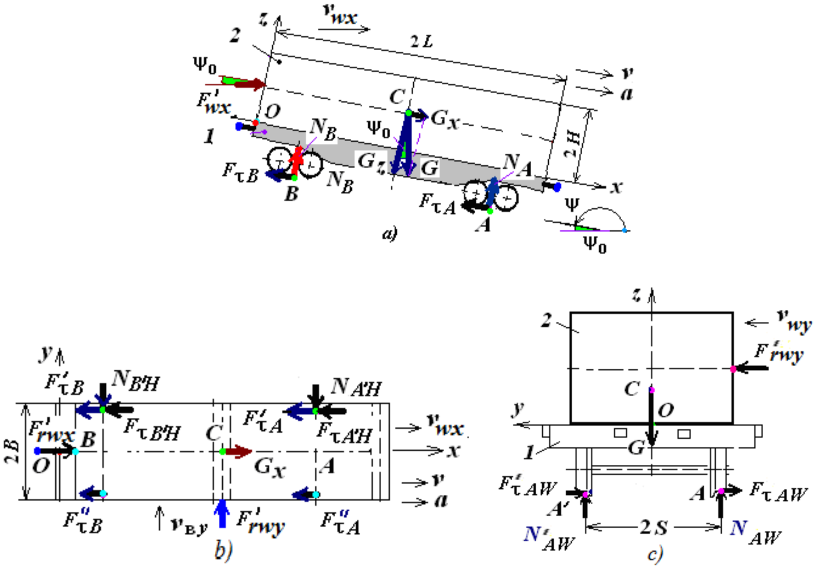

Figure 4. Design model of wagon rolling down the hump *a)*– side view; *b*) – top view; *c*) – end view.

$\overline{F}_{\tau B'} = \overline{F}_{\text{fr}B'}$, $\overline{F}_{\tau A'W} = \overline{F}_{\text{fr}A'W}$, $\overline{F}_{\tau B'W} = \overline{F}_{\tau B'W}$. It means that $\overline{F}_{\tau}$ directed along the surface of rail threads is opposite in direction to wagon rolling and is sliding friction force

$$\overline{F}_{\tau} = \overline{F}_{\text{fr}}:$$

$$\overline{F}_{\text{fr}} = \overline{F}_{\text{fr}.A} + \overline{F}_{\text{fr}.A'} + \overline{F}_{\text{fr}.B} + \overline{F}_{\text{fr}.B'} + \overline{F}_{\text{fr}.AW} + \overline{F}_{\text{fr}.A'W} + \overline{F}_{\text{fr}.BW} + \overline{F}_{\text{fr}.B'W}.$$

Friction force during wheel rolling motion with sliding $F_{\text{fr}}^{\text{sl}}(\overline{v}_e)$ as braking force of the wagon $F_{\text{brak.}}(v_e)$, directed oppositely wagon rolling down the hump is to be presented in the form (Khabibulla,2014):

$$F_{\text{brak.}}(\overline{v}_e) = F_{\text{fr}}^{\text{r}}(v_e) + f_{\text{sl0}} F'_{rwy} \quad (6)$$

where $F_{\text{fr}}^{\text{r}}(\overline{v}_e)$ is symbolic sliding friction during ideal wheel rolling and rolling elements in bearing boxes:

$$F_{\text{fr}}^{\text{r}}(v_e) = f_0\big(G\cos(\psi_{0.50}) + F'_{rwx}(v_e)\times\sin(\psi_{0.50})\big) \quad (7)$$

Here f_0 is some symbolic (or reduced) factor of sliding friction (Khabibulla,2014):

$$f_0 = \frac{n_{\text{r}} f_{\text{r}}}{r_{\text{r}}} + \frac{n_{\text{t}} f_{r0}}{r_{\text{t}}} \frac{k}{n_{\text{b}} n_{\text{mr}}}; \quad (7\text{ a})$$

f_{sl0} is a coefficient of sliding friction of wheel flanges down the rail (normally taken as – f_{sl0} = 0,25); F'_{rwy} is projections of aerodynamic resistance force onto the wagon transverse axis (according to (5) this value is calculated).In (7 a) as in (Vodnev , 1998) the following symbols are accepted: $n_{\text{к}}$ is the number of wheels in bogies, items, (n_{r} = 8); f_{r} is rolling friction coefficient, as this coefficient is omnipotent to an arm of couple of rolling friction (wheel according to rail $f_{\text{r}} = 5\times10^{-6}$, hardened steel according to steel $f_{\text{r}} = 1\times10^{-6}$); r_{r} is wheel radius equal for a freight car 0.475 m; n_{b} = 8 is the number of box units in bogies, items, is coefficient of rolling friction according to race rings (normally is taken as 0.001×10^{-3}), m; n_{tq} is a total number of rolling elements, perceiving load in each bearing, units; k is permanent coefficient taken according to row layout and type of rolling bearings (for calculation it is taken k = 4, 6) (Khabibulla,2014); is the number of bearings in bogie box units, units (n_{t} = 16); r_{t} is outer radius of internal rolling bearing ring, m (0,079 m).

Introducing the notions of "shearing" $F_{\text{shear}.x}$ and "retaining" $F_{\text{ret}.x}$ forces with consideration for active and all reactive forces we will get:

$$F_{\text{shear}.x} = G\sin(\psi_0) + F'_{rwx}(v_e)\cos(\psi_0); \quad (8)$$

$$F_{\text{ret}.x}(v_e) = F_{\text{brak.}}(v_e)$$

We rewrite the above expression with consideration for (6) and (7)

$$F_{\text{ret}.x}(v_e) = f_0\left(G\cos(\psi_0) + F'_{rwx}(v_e)\sin(\psi_0)\right) + f_{\text{sl0}}F'_{rwy}. \quad (9)$$

The condition of wagon rolling down the first profile hump section with gradient not steeper than 50‰ at section length up to 50 m is (Khabibulla, 2014):

$$F_{\text{shear}.x} >> F_{\text{ret}.x}(v_e) \quad (10)$$

It follows that excess of forces $\Delta F_{\text{r}.50} = F_{\text{shear}.x} - F_{\text{ret}.x}(v_e)$ occurring on the first hump section is the motive force causing wagon rolling of given force *G* and fair wind force $F'_{\text{wx}}(v_e)$ at speed $v_{e.50}(t)$ and acceleration depending mostly on rolling angle of the hump $\psi_{0.50}$ and to some degree of sliding friction of wheel flanges down the rail and also on the state of rolling bearing in bogie journal box units. That is why in order to ensure the wagon motion at the end of the first profile hump section at speed $v_{e.50}(t)$ less than speed $v_{ewx}(t)$ of entrance onto the first braking position (1st BP), that is $v_e(t) < v_{ewx}(t)$, it is enough to find rational value $\psi_{0.50}$ as a hump major geometrical parameter.

RESULTS OF SOLUTION

Mathematical description of the dynamics of wagon rolling down the hump

We will take into account the fact that a wagon is rolling down the hump progressively that is why wagon transferring acceleration $\overline{a}_e = \overline{a}$ is equal to absolute acceleration $\overline{a}_{\text{abs}} = d\overline{v}_{\text{abs}} / dt$ (Loitsyansky,1983).

The fundamental law of dynamics for wagon transferring movement (or principle of D'Alembert) in coordinate form (Loitsyansky,1983) was used:

$$M\frac{dv_{a.x}}{dt} = \sum_{k=1}^{n} F_{kx} + \sum_{k=1}^{n} R_{kx} \quad (11)$$

where *M* is mass of the wagon with cargo, kg; $F_x = F_{\text{shear}.x}$ is projections of all active (shearing) forces onto the direction wagon rolling (axis *x*), N; $R_x = F_{\text{ret}.x}(v_e)$ is projections of all reactive (retaining) forces onto axis *x*, N.

Substituting (11) for (8) and (9) we will get:

$$M\frac{dv}{dt} = F_{\text{shear}.x} - F_{\text{ret}.x}(v_e).$$

Transforming the above expression with consideration for (8) and (9) and the fact that *G* = *M*g for the first profile hump section with gradient not steeper than 50 ‰ at section length up to 50 m we will get:

$$M\frac{dv}{dt} = Mg\sin(\psi_0) + F'_{rwx}(v_e)\cos(\psi_0) - f_{\text{sl0}}F'_{rwy} - \\ - f_0\left(Mg\cos(\psi_0) + F'_{rwx}(v_e)\sin(\psi_0)\right). \quad (12)$$

Putting (4) in (12) for fair wind we present the fundamental law of dynamics in the form

$$M\frac{dv}{dt} = F_0 + b_0(c_0 - v_e\cos(\psi_0))^2 \quad (13)$$

where F_0 is the difference between known values of motive forces and resistance forces applied to the system "wagon-cargo", N;

$$F_0 = Mg\left(\sin(\psi_{0.50}) - f_0\cos(\psi_{0.50})\right) - f_{\text{sl0}}F'_{rwy},$$

b_0 is permanent coefficient, and, the value of which is known, having dimension N/(m/s $)^2$:

$$b_0 = 0.5c_{\text{w}}\rho_{\text{w}}A_{\text{EV}}\left(\cos(\psi_{0.50}) - f_0\sin(\psi_{0.50})\right);$$

c_0 is permanent coefficient, the value of which is known, having dimension of speed, m/s:

$$c_0 = v_{aw}\cos(\xi).$$

Designating $v_e\cos(\psi_0)$ through *v* and separating both parts (13) by b_0 we will have:

$$\frac{M}{b_0}\frac{dv}{dt} = a_0^2 + (c_0 - v)^2 \quad (14)$$

where a_0^2 is a constant value having dimension of speed, (m/s)^2: $a_0^2 = \dfrac{F_0}{b_0}$.

Separating variables (14) and fulfilling transformations we will get (Piskunov,1978):

$$\frac{b_0}{M}dt = -\frac{d(c_0 - v)}{a_0^2 + (c_0 - v)^2}$$

Taking integrals from rational functions of both parts of the above equations we will have:

$$\frac{b_0}{M}t = -\int_{v_0}^{v} \frac{d(c_0 - v)}{a_0^2 + (c_0 - v)^2}$$

The right part of the above equation is a tabulated integral from rational function in the form (Bronstein,1980 and Vodnev,1998):

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\,\mathrm{arctg}\frac{x}{a}$$

In accordance with this statement, we present the right part of the equation in the form:

$$\frac{b_0}{M}t = -\frac{1}{a_0}\,\mathrm{arctg}\frac{c_0 - v}{a_0}\Bigg|_{v_0}^{v}$$

Hence: putting integration limits v and v_0 after transformation we will get:

$$\frac{b_0}{M}t = -\frac{1}{a_0}\,\mathrm{arctg}\frac{c_0 - v_0}{a_0} + \frac{C}{a_0} \qquad (15)$$

where C is a constant number

$$C = \mathrm{arctg}\frac{c_0 - v_0}{a_0}.$$

Transforming (15) we will get the following trigonometric equation (Ilin,1967):

$$\mathrm{arctg}\frac{c_0 - v_0}{a_0} = C - \frac{b_0 a_0}{M}t$$

Multiplying both parts of the above equation by tangent:

$$\mathrm{tg}\left(\mathrm{arctg}\frac{c_0 - v_0}{a_0}\right) = \mathrm{tg}\left(C - \frac{b_0 a_0}{M}t\right)$$

Transforming the above expression after elementary computation we will have:

$$\frac{v}{a_0} = \frac{c_0}{a_0} - \mathrm{tg}\left(C - \frac{b_0 a_0}{M}t\right)$$

Because tangent (tg) is an odd function (Ilin,1967), factoring out “minus” behind tg the above expression will be written in the form:

$$\frac{v}{a_0} = \frac{c_0}{a_0} + \mathrm{tg}\left(\frac{b_0 a_0}{M}t - C\right)$$

Then we will fine the projection of wagon speed on the longitudinal axis *Ox* during wagon rolling from the hump under the impact of projection of gravity forces and fair wind onto axis *Ox:*

$$v = c_0 + a_0\,\mathrm{tg}(\alpha t - C).$$

(16)

where α (alpha) is a constant value having dimension 1/s:

$$\alpha = a_0\frac{b_0}{M}.$$

Taking into consideration the fact that previously $v_e\cos(\psi_0)$ was designated through v from (16) we will find wagon transferring speed v_e towards the direction wagon rolling down the hump O_1x_1

$$v_e(t) = \frac{1}{\cos(\psi_0)}\left(c_0 + a_0\,\mathrm{tg}(\alpha t - C)\right) \qquad (17)$$

It follows that wagon speed depending on speed during rolling down the hump profile is described according to the law (17): the wagon can quickly gain speed and in what follows in the absence of forced resistance will move practically uniformly.

Taking into account the fact that $v_e = \frac{dx_1}{dt}$ we rewrite the above expression:

$$\frac{dx_1}{dt} = \frac{1}{\cos(\psi_0)}\left(c_0 + a_0\,\mathrm{tg}(\alpha t - C)\right)$$

Multiplying both parts of the above equation by *dt* we will have:

$$dx_1 = \frac{1}{\cos(\psi_0)}\left(c_0 + a_0\,\mathrm{tg}(\alpha t - C)\right)dt$$

Integrating the obtained equation we have:

$$dx_1 = \frac{1}{\cos(\psi_0)}\left(c_0 t + a_0\int_0^t \frac{\sin(\alpha t - C)}{\cos(\alpha t - C)}dt\right) \qquad (18)$$

Then we substitute the variable in the secondly summand (18) $\alpha t - C = z$, from here $\alpha dt = dz$, hence

$$dt = \frac{1}{\alpha} dz$$

In accordance with this statement let us present (18) after transformations in the form:

$$dx_1 = \frac{1}{\cos(\psi_0)}\left(c_0 t - \frac{a_0}{\alpha}\int_0^t \frac{1}{\cos(z)} d\cos z \right) \quad (19)$$

The second member under the bracket of the above equation is a tabulated integral containing trigonometric function in the form (Bronstein,1980; Vodnev,1998; Ilin,1997):

$$\int \frac{dz}{\cos az} = \frac{1}{a}\ln\left|\mathrm{tg}(\frac{az}{2} + \frac{\pi}{4})\right|$$

According to this statement we will write (19) in the form:

$$x_1 = \frac{1}{\cos(\psi_0)}\left(c_0 t - \frac{a_0}{\alpha}\ln\left|\mathrm{tg}(\frac{z}{2} + \frac{\pi}{4})\right|\Bigg\|_0^t \right).$$

Taking into account the fact that $z = \alpha t - C$ we rewrite the above expression

$$x_1 = \frac{1}{\cos(\psi_0)}\left(c_0 t - \frac{a_0}{\alpha}\ln\left|\mathrm{tg}(\frac{\alpha t - C}{2} + \frac{\pi}{4})\right|\Bigg\|_0^t \right).$$

Substituting integration limits of the above equation after elementary mathematical computation we will get:

$$x_1 = \frac{1}{\cos(\psi_0)}\left(c_0 t - \frac{a_0}{\alpha}\ln\left|\frac{\mathrm{tg}(\frac{\alpha t - C}{2} + \frac{\pi}{4})}{\mathrm{tg}(\frac{\pi}{4} - \frac{C}{2})}\right| \right).$$

Designating $\frac{\alpha}{2} = \alpha_1$ and $\frac{\pi}{4} - \frac{C}{2} = \beta$, we present the above expression in the form:

$$x_1(t) = \frac{1}{\cos(\psi_0)}\left(c_0 t - \frac{a_0}{\alpha}\ln\left|\frac{\mathrm{tg}(\alpha_1 t + \beta)}{\mathrm{tg}(\beta)}\right| \right). \quad (20)$$

It follows that the distance (way) traveled depending on time is described according to the law (20): with increasing rolling time leads to linear increasing the distance traveled. As in (Khabibulla,2014) the method of adjustment of the solutions of piecewise-linear equations was used on the basis of Heaviside dimensionless retarded unit function (Lasaryan,2004) the speed of wagon (cut) movement and the distance travelled 20 on any hump section within the limits of the given profile up to the braking moment can be presented in the form:

$$v_e(t) = \frac{1}{\cos(\psi_0)}\left(c_0 + a_0 \mathrm{tg}(\alpha t - C)\right)\sigma_0(t - t_i) \quad (21)$$

$$x_1(t) = \frac{1}{\cos(\psi_0)}\left(c_0 t - \frac{a_0}{\alpha}\ln\left|\frac{\mathrm{tg}(\alpha_1 t + \beta)}{\mathrm{tg}(\beta)}\right| \right)\sigma_0(t - t_i). \quad (22)$$

where $\sigma_0(t - t_i)$ (where $(t_1, t_2) \in t_i$) is Heaviside dimensionless retarded unit function making it possible to present time *t* as one analytical expression fit at any value of coordinate t_i in the interval $0 \le t \le t_i$ at that at $\sigma_0(t - t_i) = 0$ at $t_i < t$.

To sum up, by means of using D'Alembert principle of mechanics, variables separation method, rational functions tabulated integral and, unlike in [5], using the integral containing trigonometric function, as well as the method of piecewise-linear equations there have been derived analytical formulas for defining the speed of wagon rolling down the hump profile $v_e(t)$ and distance traveled $x(t)$with consideration for time.

CONCLUSION

1. Obtained on the basis of classical concepts of theoretical mechanics calculated and mathematical models of the wagon rolling down the hump profile under the impact of gravity forces projection and unlike in (Khabibulla,2014) fair wind onto the longitudinal axis make it possible to define the speed of rolling wagon $v_e(t)$ and the distance traveled $x(t)$ down the first hump profile according to time. In a particular case the obtained analytical expressions of wagon rolling dynamics enable specialists to find finite formulas for defining the speed and the distance traveled either under the impact of only gravity forces projection onto the longitudinal axis or under the impact of only fair wind.
2. The results of analytical investigations of the wagon rolling down the first high speed hump section can be used for all the rest hump sections with consideration for specifics of braking forces on these sections.The distinctive feature (novelty) of the derived analytical formulas of wagon speed

rolling down the high speed hump section consists in presenting the force unlike in (Komarov,2004) of fair wind depending on wagon rolling speed $v_e(t)$ and correct consideration for resistance forces occurring the motion of the wagon. The obtained research results available for hump designers are a new stage in the solution of this particular problem. The advantage (significance) of this methodology is the possibility of the construction of mathematical model of wagon rolling down the hump with consideration for the dependence of fair wind force on wagon rolling speed $v_e(t)$, speed $\overline{v}_{a.\text{в}}$ and air flow direction (ξ).

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