Direct RESEARCH JOURNALS

Direct Research Journal of Agriculture and Food Science
Vol. 8 (10), Pp. 346-350, October 2020
ISSN 2354-4147
DOI: https://doi.org/10.26765/DRJAFS9037615271
Article Number: DRJAFS9037615271

https://directresearchpublisher.org/drjafs/

Review Paper

Mathematical Modelling of Infectious Diseases on *Hevea Brasiliensis* (Natural Rubber)

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Received 15 August 2020; Accepted 23 September, 2020

ABSTRACT: In this paper, we present an efficient method of mathematical modelling for the infectious diseases on Natural Rubber using the Kermack and Mckendrick formulated *SIR* (SUSCEPTIBLE-INFECTIVE-RECOVERED) techniques. Firstly, we examined the spread of an epidemic disease based on compartmental structures and thereby estimated the value R_0 which determines the thresholds for disease transmission and the control. We also introduce an approximate method and used the *SIR* model to ascertain the maximum level of infection.

Keywords: Plant disease epidemics, mathematical modelling, Infective, Recovered, Susceptible and Transmission

INTRODUCTION

Presently plant diseases, weeds and environmental factors are the major threats to agricultural production, mainly in developing countries as it contributes an average of 36.3 % of worldwide crop losses (Van den Bosch *et al.,* 2006). It has been taken into account that about 14.1 % of crops are lost to plant disease alone, and the total worldwide crop loss from plant diseases is about $220 billion dollars which poses greater threat to the agricultural sector and brings several problems in other important sectors such as health, environmental and economy (Agrios, 2005). Strange and Scott, (2005) reported that these losses are partly responsible for the suffering of over 800 million people lacking adequate food for consumption. In view of the above mentioned problems it is necessary to have adequate, economic and environmentally acceptable strategies to manage epidemic development of plant diseases in order to reduce the crop losses and minimize their consequences (Van Maanen and Xu, 2003; De Wolf and Isard, 2007). Most current practices have been used in order to control epidemics, like the use of chemical control (Blaise *et al.,* 1999) having a direct environmental impact due to its chemical residues (Orlandini *et al.,* 2008). Therefore, in order to obtain sustainable practices for strategic and tactical management of diseases and also to decrease its environmental impact on any crop, it is expedient to understand the determining factors of epidemics (Royle and Ostry, 1995; Jeger, 2004). Mathematical and statistical models with other tools have been used to understand these factors by modelling the epidemic dynamics. The objective in modelling is to simplify the reality in order to summarize the process of the epidemic (Van Maanen and Xu, 2003). Mathematical models have been gaining more recognition since it allows knowing a description of the epidemic dynamics and by significance, to develop optimal forecasting and controlling mechanisms. Descriptions of temporal disease progress were used prior to 1960s when Ware *et al.,* (1932) and Ware and Young, (1934) presented curves to illustrate the effects of crop resistance and fertilizer treatment for the dynamics of cotton wilt. Large (1945, 1952) proposed disease progress curves to demonstrate the benefits of fungicide applications on the development of potato late blight. Nevertheless, the first temporal development model of plant disease epidemic was proposed by Van der Plank (1960, 1963), which has been the base of many epidemiological models created so far.

The mathematical tools employed in plant disease epidemiology use several variable values as inputs; these variables are considered according to the nature of the problem and the objective questions to be achieved (Van der Plank, 1982). Schoeny *et al.,* (1999) proposed a predictive model of Ascochyta blight where an important variable in the model was the airborne inoculums. The variables used by the mathematical tools summarize the key characteristics in the epidemic dynamics. The most common mathematical tools used in plant disease epidemiology includes: disease progress curves, linked differential equations, area under disease progress curve and mathematical simulations. Nevertheless, there are other tools to evaluate the disease progress. Natural Rubber (*Hevea Brasiliensis*) as one of the world's most important commodity is also prone to such epidemic diseases. Rubber cultivation in Nigeria began over five (5) decades with over 200,000 hectares planted with natural rubber in the tropical rain forest regions of the southern states of Nigeria (Umar *et al.,* 2011). Hence, the interaction of rubber with plants and micro-organisms in the soils results to the high frequency of disease infection in the rubber plantations globally. Among 40 known diseases of *Hevea Brasiliensis* (NR) about a dozen are of worldwide importance. Rubber clones are screened for resistance to certain diseases before being recommended for large scale planting. Rubber diseases may be classified according to parts they affect such as leaf, tapping panel and root. Root diseases spread by root contact with a source of inoculum such as infected woody debris in the soil. The most significant root disease is white root rot where the pathogens travel both internally and externally along the roots, which decay in the process and ultimately influence the collar and tap root where upon the trees soon blows up (Chee,1990; Leonard and Ashley, 2011). A modern line of attack to white root disease control is to saturate soak the soil around the tree with a fungicide. Secondly, the wound exposed by tapping can be infected by several pathogenic fungi among which species of phytophthora are by far the most important Wastie, (1975) while the black stripe leaves large wounds with black lines in the tapping cut and clogs the latex flow (Omorusi, 2012). This paper presents an efficient method of mathematical modelling for the infectious diseases on Natural Rubber using the Kermack and Mckendrick techniques. The rest of the paper is organized as follows: Section 2 presents the Kermack-Mckendrick *SIR* Compartmental Model. Section 3 is the application of the techniques and Section 4 concludes the paper.

Kermack - Mckendrick Compartmental Model

Dynamic models for infectious diseases are mostly based on compartment structures as proposed by Kermack and Mckendrick. To formulate a dynamic model for the transmission of an epidemic disease on *hevea brasiliensis* (NR), the population in a given region is often split into several compartments. In the compartment model, the population is divided into three (3) compartments; a susceptible region, labeled *S*, in which all the rubber trees are susceptible if they contact with a disease; an infected compartment, labeled *I*, in which all the rubber trees are infected by the disease and infectious; and a removed compartment, labeled *R*, in which all the rubber trees are removed or recovered from the infection (Zhien and Jia, 2009).

Definition

Let the number of rubber trees in the hecterage per compartment be denoted as *S,I* and *R* at time *t*, as *S(t)*, *I(t)*, and *R(t)*, respectively. Then, the following assumptions are obtained:

(i) The disease spreads in a closed environment; that is, there is no emigration or relocation nor immigration or settlement, and neither birth nor death in the population, so that the total population remains a constant k, Â t, such that:

$$S(t) + I(t) + R(t) \equiv K \qquad (1)$$

(ii) The number of susceptible that are infected by an infected-rubber tree per unit time, at time *t*, is proportional to the total number of susceptible with the proportional coefficient (transmission coefficient) ß so that the total number of newly infectives, at time *t*, is $\beta S(t)I(t)$.

(iii) The number of recovered individual rubber trees from the infected compartment per unit time is *I(t)* at time *t*, where is the recovery rate coefficient, and the recovered individual rubber trees gain permanent immunity (Zhien and Jia, 2009).

$$\boxed{S} \xrightarrow{\beta SI} \boxed{I} \xrightarrow{\gamma I} \boxed{R}$$

We therefore, generate the corresponding model equations based on the assumptions stated above in the following systems

$$(1.2) = \begin{cases} \frac{dS}{dt} = -\beta SI \\ \frac{dI}{dt} = \beta SI - \gamma I \\ \frac{dR}{dt} = \gamma I \end{cases}$$

In equation 1.2(iii), the variable R is decoupled from the first two equations of equation 1.2(i) and (ii), then we consider

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I \qquad 1.3$$

Further simplification of (1.3) yields

$$\frac{dI}{dS} = -1 + \frac{\rho}{S} \qquad 1.4$$

where $\rho = \frac{\gamma}{\beta}$, =

It therefore follows that there exists a threshold $s = \rho$ such that if the initial number of susceptible $S(0) = S_0 > \rho$, the number of infectives increases; if $S(0) = S_0 < \rho$, the number of infectives decreases. Next, we define

$$R_0 = \beta \frac{1}{\gamma} S_0 = \frac{S_0}{\rho} \qquad 1.5$$

then the disease spreads when $R_0 > 1$ and dies out when $R_0 < 1$. Hence, we note that γI is the number of recovered individual rubber trees that were move from the infected compartment *I*, per unit of time, at time *t*, whereas after the period of time $\frac{1}{\gamma}$ all the infectives *I(t)* recovered. Therefore, $\frac{1}{\gamma}$ is the actual mean duration of infection, and $R_0 = \beta \frac{S_0}{\gamma}$ is the number of newly infectives infected by an infected individual rubber trees during the whole infection period when all the rubber trees in the population are initially susceptible. This quantity R_0 determines the thresholds for disease transmissions. The number of infectives decreases if $R_0 < 1$ and increases if $R_0 > 1$. Consequently, to control the spread of an epidemic diseases on the hecterage of *Hevea Brasiliensis*, we need one of the key factors which is to estimate the value of R_0 and then reduce it to less than 1. The estimation of R_0 involves some biological parameters which may not be easily quantified and as a result we introduce an approximate method and use the model governed by equation (1.2) to illustrate the method by integrating equation (1.4) with the initial values of (S_0; I_0) to obtain

$$I - I_0 = -(S - S_0) + \rho \ln \frac{S}{S_0} \qquad 1.6$$

It therefore follows from the second equation of (1.3) that $I(t) \to 0$, as $t \to \infty$, if $R_0 < 1$. Then since *S(t)* is monotone decreasing and bounded below, $\lim_{t\to\infty} S(t) = S_\infty$ Setting $S_0 + I_0 = K$ and taking the limit in (1.6), we have

$$I = S_0 + I_0 - S_\infty + \rho \ln \frac{S_\infty}{S_0}$$

$$I = K - S_\infty + \rho \ln \frac{S_\infty}{S_0}$$

$$0 = K - S_\infty + \rho \ln \frac{S_\infty}{S_0} \qquad 1.7$$

Algebraically, equation (1.7) yields

$$\rho = \frac{K - S_\infty}{\ln S_0 - \ln S_\infty} \qquad 1.8$$

Assume that S_0 and S_∞ are measured experimentally, then the quantity

ρ can be determine by (1.8) and then R_0 from $R_0 = \frac{S_0}{\rho}$.

If the average infection period $\frac{1}{\rho}$ is estimated, then the transmission coefficient β can also be determined by $\beta = \frac{\gamma}{\rho}$ (Zhien and Jia, 2009).

Applications

In a rootstock nursery of RRIN, Benin suffered an outbreak of root leaf diseases in 2010-2011. Information gathered shows that the initial number of susceptible and infective were 18, 545 and 12, 263 in the mid of May 2011 respectively, only 16, 326 rubber budded stumps survived in the middle of November 2011 (Omorusi, 2012). Considering the parameters in (1.8), we therefore estimate the value of ρ as follows: S_0 = 18, 545, S_∞ = 12, 326, K = 30, 871. Then substituting into equation (1.8) yields the estimated value of ρ as ρ = 46; 362 and the value of $R_0 = \frac{S_0}{\rho}$ as 0:4 which show the decline of the infection. However, the information gathered also shows that the infective period lasted for 210 days such that

$$\beta = \frac{\gamma}{\rho} = 0.005 \qquad 1.9$$

It therefore follows from (1.4) that the number of infectives I reaches the maximum as $S = \rho$. Thus from (1.6), we estimate the number of the infectives at the peak of the epidemic to be:

$$I_m = K - \rho + \rho \ln \frac{\rho}{S_0}$$

$$I_m = 27162$$

Concluding Remarks

Using the KerMack and McKendrick *SIR* technique as a method, we presented in this paper a mathematical modelling for examining the spread of infectious diseases on a rootstock nursery of natural rubber. In this type of model, the infective obtain permanent immunity to the disease after recovered from infection. We anticipate this short paper will prompt some subsequent works on this technique. The epidemiologist can chose the mathematical model which solely or jointly with other different mathematical models gives a better depiction of the reality in order to obtain a more accurate evaluation of the tree crops disease epidemics.

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