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Full Length Research Paper

Application of Mathematical Modelling in Estimation of Oil Production

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ABSTRACT: This work presents an application of mathematical modelling in the estimation of oil production in a given period. The model was applied to analyze oil production from the year 2013 to 2017 using the data obtained from NNPC, Nigeria Library. Graphical analysis of the volume of oil produced during fluid injection (V_o) and volume of oil after production (V_{oil}) within the period are presented. The Result shows that the volume of oil produced during fluid injection (V_o) and the volume of oil after production in 2017, (V_{oil}) is the same.

Keywords: Mathematical modelling, oil production, volume of oil

INTRODUCTION

A mathematical model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed mathematical modelling. Mathematical models are used not only in the natural sciences (such as physics, biology, earth science, meteorology) and engineering disciplines (Livinus et al., 2014).

In many cases, the quality of a scientific field depends on how well the mathematical models were developed on the theoretical side agree with results of repeatable experiments. Lack of agreement between theoretical mathematical models and experimental measurements often leads to important advances and helps in the development of better theories.

Muonagor and Nnakaihe, (2013) in their work introduced simple and direct equations for the computation of volume of oil recoverable from a reservoir before, at and after breakthrough during production by any kind of fluid injection.

Before then, there had not been any specific model or equation for computing oil recovery during all kinds of fluid injection. Estimation of oil recovery was simply based on taking certain mathematical steps, hypothetical assumptions and speculations (Muonagor and Nnakaihe, 2013). Then they introduced simple and direct equations for the computation of volume of oil recoverable from a reservoir before and at breakthrough during production by any kind of fluid injection. The equation is based on the rules of expressing viscosities and energy in terms of pressure and volume.

In developing the equations for the estimation of the volume of oil produced from a reservoir before, at and after breakthrough during enhanced oil recovery by fluid injection, certain parameters were taken into considerations. The parameters are given below:

(i) The flood area (A)
(ii) Reservoir thickness (h)
(iii) Injection time
(iv) Production time
(v) Oil formation volume factor (Bo)
(vi) Oil viscosity (α)

(vii) Injected fluid viscosity (β)
(viii) Injection rate
(ix) Reservoir porosity (Φ)
(x) Connate water saturation (Swc)

Livinus et al. (2014) presented an equation, based on reconciliation factor, for estimating crude oil theft volume, and a model for crude oil theft volume-share among oil producers utilizing a pipeline network belonging to a major producer.

The model formulated tried to allocate crude oil stolen volume proportionally to all flow stations and injectors connected to a pipeline network pro rata their injection and pipeline exposures. The formulated model also shared the loss in a manner that consider the varying volume of the "dry crude" and/or "wet crude" delivered into the oil transport pipeline network. They illustrated the applicability of the models with Data from an X terminal pipeline network, experiencing crude oil theft, in the Niger Delta.

Many other authors have worked on this area of research, among them are, Cheng and Shen, (2014) who worked on the Mathematical Modelling of Crude Oil using Gas-lift. Also, Cheng (2017) worked on the mathematical modelling of petroleum extraction processes and Eghon and Idigbe, (2013) on the complexity of oil extraction. The present work is an application of the mathematical model developed by Muonagor and Nnakaihe, (2013) in estimating the volume of oil within a given period.

Equations for the estimation of the volume of oil produced before, at and after breakthrough from a reservoir produced by fluid injection using gas-lift Muonagor and Nnakaihe, (2013)

Estimation of the volume of oil production before breakthrough

From fluid mechanism, Energy = Pressure * Volume
Therefore, Energy of Injection fluid, $E_i = P_i * V_i$
And Energy of Produced fluid, $E_p = P_p * V_p$
Where P_i, V_i, P_p, V_p are injection fluid pressure, injection fluid volume, produced fluid pressure and produced fluid volume respectively.
E_i greater than or equal to E_p for flow to occur. That is,
$P_i V_i \geq P_p V_p$

When
$P_i V_i = P_p V_p$ then V_i is the minimum volume of fluid in barrel injected at pressure P_i (psi) to cause a volume of V_p of oil (bbl) to be produced at pressure P_p (psi).
According to Muonagor and Nnakaihe, (2013) the volume of oil (bbl) is expressed as:

$$V_{oil} = \frac{7758Ah\Phi(1-Swc)}{Bo}$$

Where B_o = oil formation factor, rb/stb

Then, the minimum volume of injection fluid required to displace V_{oil} is given as:

$$V_i = P_p * \frac{V_{oil}}{P_p}$$

Which is the same as:

$$V_i = \frac{P_p * 7758Ah\Phi(1-Swc)}{P_i * Bo}$$

The volume of oil before breakthrough can be expressed as:

$$V_o = \frac{[t_i t_o \beta (I_{rate})^2 Bo]}{[7758Ah\Phi\alpha(1-Swc)]}$$

Where t_i and t_o are in days; β and α in cp; I_{rate} in bbl/day; B_o in rb/stb; A in acre; h in ft.

Estimation of the volume of oil production at breakthrough

V_{ob} = 7758AhΦ * E*Q
Where E is Area Sweep Efficiency at breakthrough and Q is Pore Volume of injected fluid at breakthrough.
V_{ob} can also be expressed as:
$V_{ob} = I_{rate} * T_i$

Where T_i is injection time to breakthrough in day.
Therefore, the volume of oil produced at breakthrough in barrel is expressed as:

$$V_b = \frac{[7758Ah\Phi(EQ)^2 T_i \beta Bo]}{[T_i \alpha(1-Swc)]} \quad (1)$$

(2)

T_i is oil production time to breakthrough in day.

Estimation of the volume of oil production after breakthrough

(3)

The volume of oil after breakthrough is at a larger amount compare to the volume of oil before and at breakthrough as the oil is the mixture of both the oil and some of the injected fluid. This eventually led to the separation of the mixture to get the exact oil produced.

$$V_a = V_b + V_f - F_p$$

Table 1: Data of oil production from the year 2013-2017 in Nigeria.

Parameters	2013	2014	2015	2016	2017
A (acres)	10	20	30	40	50
h (ft)	20	40	60	80	100
t_i (days)	200	300	400	500	600
t_o (days)	180	280	380	480	580
B_o (rb/stb)	1.401	1.401	1.401	1.401	1.401
α (cp)	1	1	1	1	1
β (cp)	1.9	1.9	1.9	1.9	1.9
I_{rate} (bbl/day)	4000	4200	4400	4600	4800
Φ	0.2	0.2	0.2	0.2	0.2
S_{wc}	0.2	0.2	0.2	0.2	0.2

Source: Nigerian National Petroleum Development Company, NNPC.

Table 2. Analysis of oil production from the year 2013 to 2017 with the volume of oil produced during fluid injection (V_o) and volume of oil after production (V_{oil}) respectively.

Parameters	2013	2014	2015	2016	2017
A (acres)	10	20	30	40	50
h (ft)	20	40	60	80	100s
t_i (days)	200	300	400	500	600
t_o (days)	180	280	380	480	580
B_o (rb/stb)	1.401	1.401	1.401	1.401	1.401
α (cp)	1	1	1	1	1
β (cp)	1.9	1.9	1.9	1.9	1.9
I_{rate} (bbl/day)	4000	4200	4400	4600	4800
Φ	0.2	0.2	0.2	0.2	0.2
S_{wc}	0.2	0.2	0.2	0.2	0.2
V_o	1,998,102.088	2,872,005.655	3,397,891.037	3,403,289.588	3,438,853.648
V_{oil}	1,207,199.143	1,501,796.573	1,594,792.291	3,438,853.643	3,429,978.587

Where

V_f is the total subsequent fluid production after breakthrough in barrel

F_p is the injection fluid production in barrel

V_b is the volume of oil produced at breakthrough in barrel

In general, the volume of oil produced in barrel is expressed as:

$$V_{oil} = \frac{7758Ah\Phi(1-Swc)}{Bo} \quad (12)$$

The following (Tables 1) gives the data analysis of oil production from the year 2013 to 2017 in Nigeria according to NNPC, (2017).

Application of the Model

Analysis of data using the mathematical model

Table 2 shows the estimation of the volume of oil production from a reservoir during injection in barrel is:

$$V_o = \frac{[t_i t_o \beta (I_{rate})^2 Bo]}{[7758Ah\Phi\alpha(1-Swc)}$$

And in general, the volume of oil produced in barrel is;

$$V_{oil} = \frac{7758Ah\Phi(1-Swc)}{Bo}$$

Therefore, for 10 acres of flood area as at 2013;

$$V_o = \frac{[200*180*1.9*(4000)^2*1.401]}{[7758*10*20*0.2*0.8]}$$

= 1,998,102.088 bbl

$$V_{oil} = V_{oil} = \frac{(7758*10*20*0.2*0.8)}{(1401)}$$

= 1,207,199.143 bbl

For 20 acres of flood area as at 2014;

$$V_o = \frac{[300*280*1.9*(4200)^2*1.401]}{[7758*20*40*0.2*0.8]}$$

= 2,872,005.655 bbl

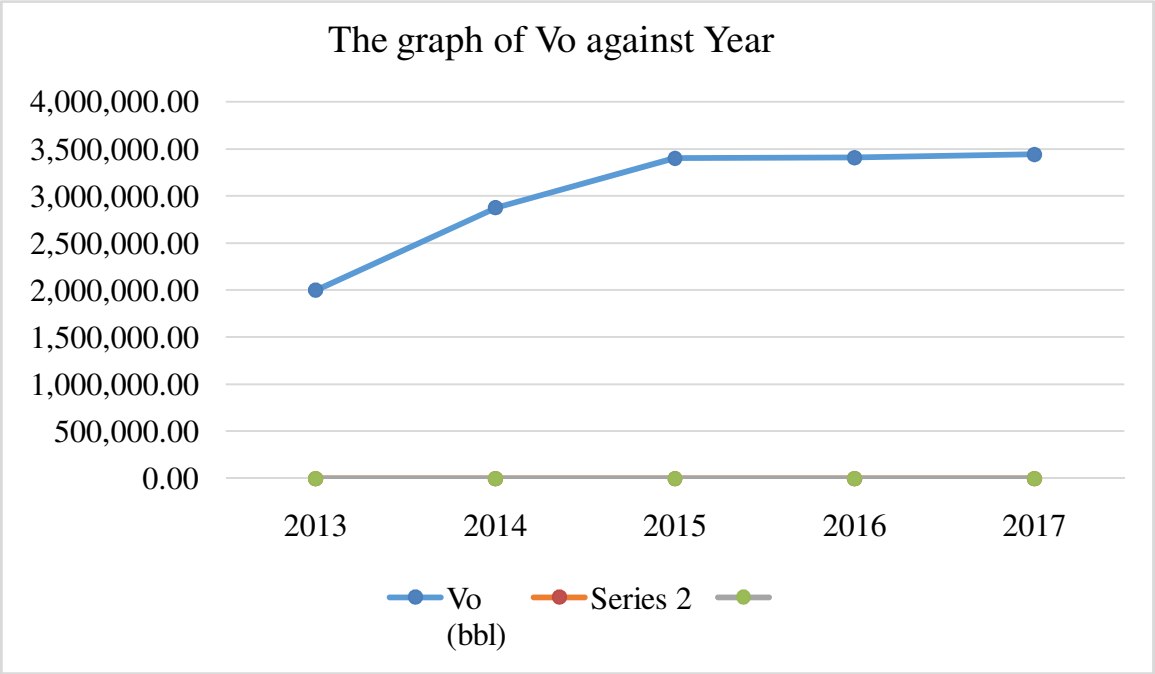

Figure 1: The Graphical Representation of the Volume of Oil Produced during fluid injection (V_o).

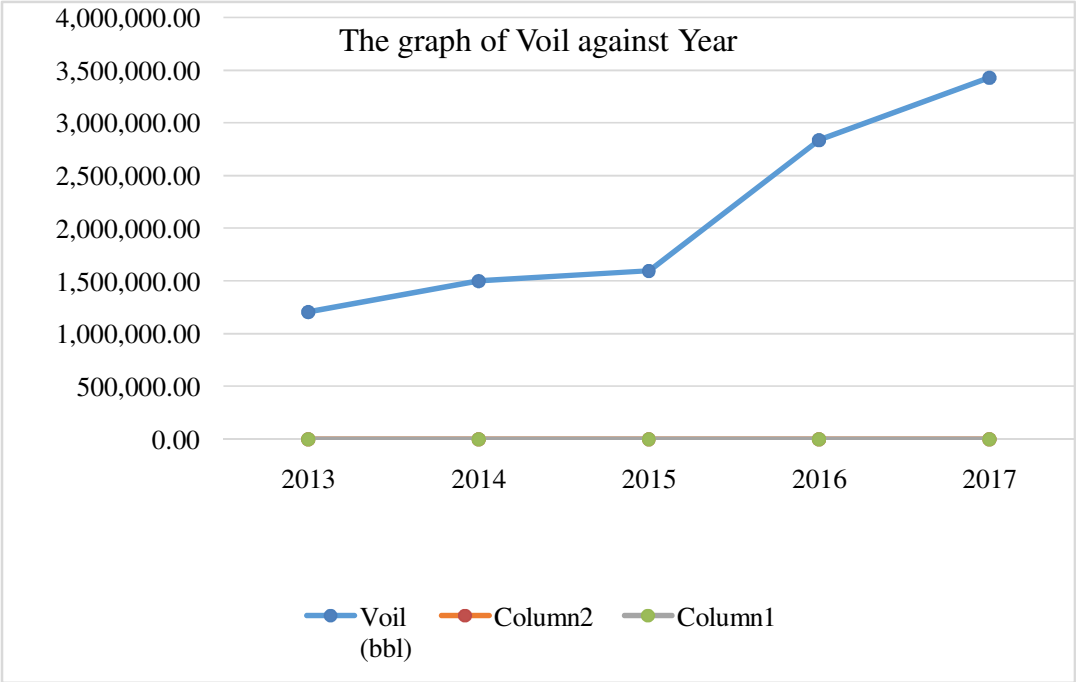

Figure 2: Volume of oil produced after production (V_{oil}).

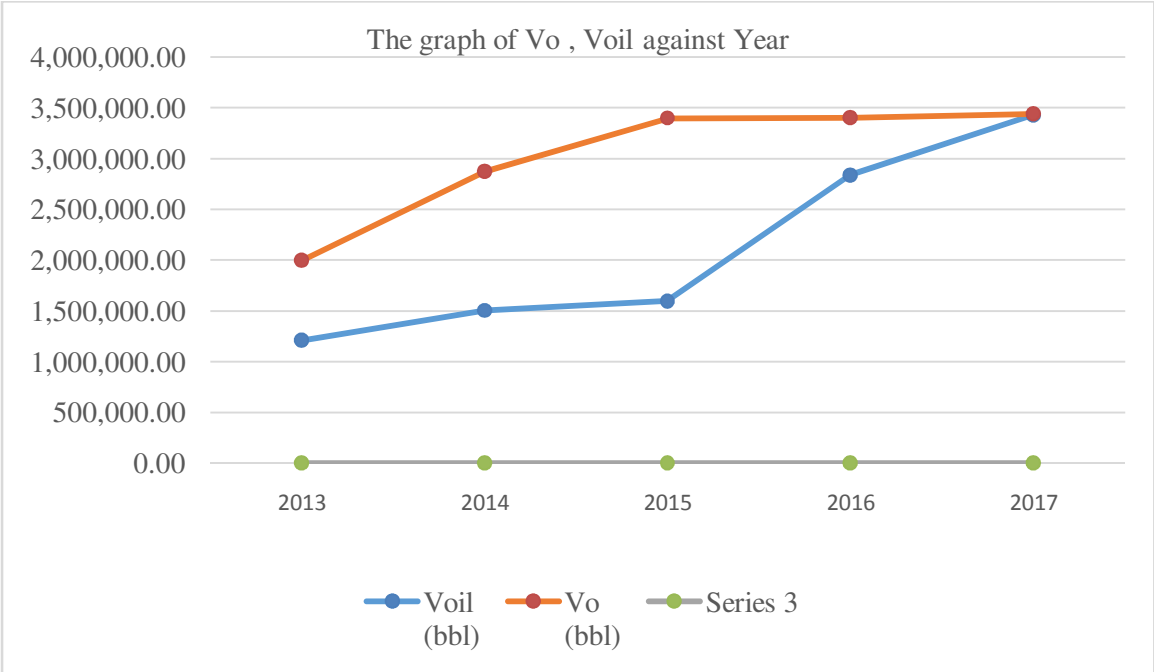

Figure 3: Volume of oil produced during fluid injection (V_o) and volume of oil after production (V_{oil}).

$$V_{oil} = \frac{[7758 * 20 * 40 * 0.2 * 0.8]}{(1.401)}$$

= 1,501,796.5739 bbl

For 30 acres of flood area as at 2015;

$$V_o = \frac{[400 * 380 * 1.9 * (4400)^2 * 1.401]}{[7758 * 30 * 60 * 0.2 * 0.8]}$$

= 3,397,891.037 bbl

$$V_{oil} = \frac{(7758 * 30 * 60 * 0.2 * 0.8)}{(1.401)}$$

= 1,594,792.291 bbl

For 40 acres of flood area as at 2016;

$$V_o = \frac{[500 * 480 * 1.9 * (4600)^2 * 1.401]}{[7758 * 40 * 80 * 0.2 * 0.8]}$$

= 3,403,289.588 bbl

$$V_{oil} = \frac{(7758 * 40 * 80 * 0.2 * 0.8)}{(1.401)}$$

= 2,835,186.296 bbl

For 50 acres of flood area as at 2017;

$$V_o = \frac{[600 * 580 * 1.9 * 4800^2 * 1.401]}{[7758 * 50 * 100 * 0.2 * 0.8]}$$

= 3,438,853.643 bbl

$$V_{oil} = \frac{(7758 * 50 * 100 * 0.2 * 0.8)}{(1.401)}$$

= 3,429,978.587 bbl

RESULTS AND DISCUSSION

The volume of oil produced during fluid injection (V_o) and volume of oil after production (V_{oil}) have been analyzed for the year 2013 to 2017 using the data from NNPC, library by the mathematical model (Table 2). Figures 1 to 3 show that in 2017, the volume of oil produced during fluid injection (V_o) and the volume of oil after production (V_{oil}) is the same.

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