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Full Length Research Paper

Application of Nonlinear First Order Ordinary Differential Equation Model in Stock Market Analysis

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Abstract: This paper is on the application of nonlinear first order ordinary differential equation model in stock market analysis. The data used is from the Nigeria Stock Exchange (NSE), Stock Market Report for September, October and November, 2019 and the Stock Share Index from 2011-2016. The model used considers two competing growth rates and its carrying capacities. The analysis shows a continuous growth in the stock strength within the period considered.

Keywords: Ordinary differential equation model, stock market

INTRODUCTION

Differential equation is one of the most frequently used tools for mathematical modelling in life sciences and engineering. Generally, dynamics of changing process are modelled into differential equations (Dennis, 1979). However, financial data are gaining popularity in mathematical modelling. A lot of research work has been carried out in the mathematical modelling of stock market variables.

Tian-Quan and Yun, (2009) established a set of simultaneous differential equations of stock prices of the same share in both A and H stock markets. The set of simultaneous nonlinear differential equations could be solved by iteration method via a proof by g-contraction mapping theorem. Ren, Cheng et al. (2011) proposed a novel approach called Dynamic Ordinary Differential Equation (DyODE) modelling of stock market prediction with Gene Expression Programming (GEP). DyODE selects suitable data as training set to build prediction model and hence avoids prediction error caused by using obsolete data. Prediction accuracy of DyODE was tested on the stock prices series. Their results showed that the accuracy is much higher than that of traditional approaches.

Charlene C. de Resende et al. (2017) proposed a new differential equation based model for stock price trend forecast as a tool to investigate efficiency in an emerging market. Its predictive power showed statistically to be higher than the one of a completely random model, signaling towards the presence of arbitrage opportunities. They also investigated the conditions for accuracy to be enhanced and discussed application of the model as part of a trading strategy. Meng-Rong et al. (2018) considering the phenomenon of the mean reversion and the different speeds of stock prices in the bull market and in the bear market, proposed four dynamic models each of which was represented by a parameterized ordinary differential equation. They solved the models by dynamic integration and applied them to the daily closing prices of the Taiwan stock index, Taiwan Stock Exchange Capitalization Weighted Stock Index. Their study showed that some of the models fit the prices well with an acceptable forecasting ability.

Amadi et al. (2017) worked on the application of nonlinear first order ordinary differential equations on stock market prices. Their paper modelled two competing growth rates and its carrying capacity on stock market prices which was chosen based on minimum variance criteria. They discovered within the trading period that resources to sustain the consumers (population) are limited. Many other authors have worked on this interest; among them is Sirajo (2017) and Adeosun et al (2015.) This paper adopts the nonlinear ordinary differential equation model developed in Amadi et al. (2017) to analyze the all share index of the Nigerian Stock Exchange within a particular period.

METHODOLOGY

The stock market

The transaction which takes place between the two parties is known as trade in stock market. There is a transfer for money of a stock or security from a seller to the buyer. The participants in the stock market varies, it can be small stock investors to large trade investors. These investors can be based anywhere in the world, they may include banks, insurance companies and hedge funds. Trading is done on the most popular exchanges as it gives the best price and the largest number of potential counter parties i.e. buyers for a seller and sellers for a buyer. According to Lee G. Cooper and Masao Nakanishi, (2010), the concept of market shares may be stated in the following manner

$$S_i = \frac{Q_i}{Q}$$

Where

S_i = the market share of firm i

Q_i = the sales of firm i's product

Q = the total sales for the market

$$Q_i = \sum_{j=1}^{m} Q_j$$

m = the number of competing firms

market price and model formulation

According to Amadi et al. (2017) let $S_i(t)$ be the price of the i^{th} stock at time t (time is counted for trading days in multiples of fundamental units, say days). Thus, let an N×n data matrix, associated with $S_i(t)$ be X_{ij}. We therefore consider N stock prices over n week time horizon. For each X_{ij}, we define the vector K_{it} such that

K_{it} = (K_{it}, K_{i1}, K_{i2}, K_{i3},……., K_{in}) (1)

The two positive intrinsic growths of stock prices were chosen base on minimum variance criterion. Let denote N_1 the stock price with most minimum variance and N_2 the one with second most minimum variance in other to achieve our purpose in the work, they formulated Lokta Volterra Equations of the form:

$$\frac{dN_1(t)}{dt} = N_1(t)(a_1 - b_{11}N_1(t) - b_{12}N_2(t)) \quad (2)$$

$$\frac{dN_2(t)}{dt} = N_2(t)(a_2 - b_{21}N_1(t) - b_{22}N_1(t)) \quad (3)$$

With initial conditions

$N_1(0)=N_{10}, N_2(0)=N_{20}$ (4)

Where N_1 (t) , N_2 (t) denote the growth rates of two respective interacting stock prices, a_1, b_{11}, a_2, b_{22}, are the logistic parameters for the first and second stock price and b_{12}, b_{21} arise from the inhibiting effect on the growth of the first stock price due to the presence of the first stock prices. For the purpose of this formulation an assumption was carried out: we assume that the two competing growth rates would grow logistically in the absence of the other.

Furthermore, we define growth rate as:

Growth rate = $\frac{Present - Past}{Past}$ which is represented as:

$r_1 = \frac{K_{12} - K_{11}}{K_{11}}$ for the first stock price (5)

Similarly

$r_2 = \frac{K_{22} - K_{21}}{K_{21}}$ for the second stock price (6)

b_{11} and b_{21} are given as follows: $b_{11} = \frac{a_1}{k_{1n}}$

$$b_{21} = \frac{a_2}{k_{2n}} \quad (7)$$

Where k_i is the carrying capacity (the minimum size that can sustain a given population) with the best two stock prices as the case may be. Table 1 and Table 2 show the Stock Share index on Nigeria Stock Exchange from 2011-2016 and the weekly stock price from the period of 6/9/19 to 15/11/19. We apply the above Lokta Volterra equations (2) and (3) to carry out the mathematical model of finding the growth rates and carrying capacity of two interacting stocks.

Analysis of the stock market prices

The monthly analysis of the stock market prices

The following are calculated from the stock price data:

Table 1. The All Share index on Nigeria Stock Exchange from 2011-2016.

MONTHS	2011	2012	2013	2014	2015	2016
JANUARY	23916.2	20875.8	31853.2	34657.2	28642.1	26830.7
FEBUARY	24570.7	33075.1	39558.9	30103.8	20133.5	26016.8
MARCH	24621.2	20652.5	33536.3	38748	31744.8	25306.2
APRIL	25041.7	22045.7	33440.6	38492.1	34708.1	27663.2
MAY	25886.6	22066.4	37794.8	41474.4	34310.4	27663.2
JUNE	24980	21599.6	36164.3	42482.5	33456.8	29597.8
JULY	23827	23061.4	37914.3	42097.5	30180.3	28009.9
AUGUST	21497.6	23750.8	36248.5	41532.3	29684.8	27599
SEPTEMBER	20373	26011.6	36585.1	41210.1	31214.8	28335.4
OCTOBER	20935	26430	37622.7	37550.2	29177.7	27720.1
NOVEMBER	20003.4	26494.4	38920.9	34543.1	27617.5	25333.4
DECEMBER	26874.6	28730	41329.2	40571.6	28642.3	28078

Table 2. The weekly stock price from the period of 6/9/19 to 15/11/19.

6/09/19	13/09/19	20/09/19	04/10/19	11/10/19	18/10/19	25/10/19	01/11/19	08/11/19	15/11/19
0.06	0.51	0.09	0.38	0.1	0.05	0.06	0.28	0.13	0.17
0.11	0.09	2.25	0.05	0.48	0.4	0.04	0.07	1.05	0.13
1.05	6.3	7.85	0.02	1.65	0.55	0.07	0.07	1.4	5.3
0.11	2.05	0.37	2.3	0.11	0.03	0.14	0.4	0.26	0.13
0.18	0.85	1.3	0.2	0.04	0.03	0.02	4.8	0.8	1.05
1.55	0.03	1.15	1	0.6	0.1	0.07	0.04	3.4	1.75
0.11	1.6	1.45	0.02	0.04	0.12	0.09	0.5	0.1	0.85
0.03	0.06	6.55	0.12	0.02	2.5	0.01	0.05	0.09	0.08
0.04	1.3	0.05	0.03	0.09	0.65	0.01	0.5	0.5	0.15
0.6	4.25	0.06	0.02	0.03	0.07	0.08	0.2	0.04	1.2

The carrying capacity (CC): the minimum size of the above stock prices.

(i) Growth rate: represented as r
(ii) Range: the highest value of the stock price
(iii) Mean: the average value of the stock price
(iv) Variance: is the measurement of the spread between the stock price, it measures the vitality from an average or mean and vitality is a measure of risk.
(v) Standard deviation (SD): is the square root of variance a_1, b_{11}, a_2, b_{22}, are the logistic parameters for the first and second stock price and b_{12}, b_{21} arise from the inhibiting effect on the growth of the first stock price due to the presence of the first stock prices as defined earlier.

The computation of the key parameter values of the model is shown in (Table 3). From Table 3, the two stock prices with the smallest/second to the smallest are the 1st and 10th stock prices (SD = 5062.147, Variance = 2.6E+07) and (SD = 6570.41, Variance = 4.3E+07) respectively, where 34657.2 and 37622.7 are the carrying capacity of the first and second best stock prices. On the basis of these estimated parameter values and simplifying assumptions, we now obtain the full Mathematical structure as follows:

$$\frac{dN_1(t)}{dt} = 0.12186(a_1 - 3.516*10\text{^}(-6)N_1(t) - b_{12}N_2(t))$$

$$\frac{dN_2(t)}{dt} = 0.32410(a_2 - 8.615*10\text{^}(-6)N_1(t) - b_{22}N_1(t))$$

and $N_1(0) = 100$, $N_2(0) = 150$

We input these data into the model and solve using the maple software.

The weekly analysis of the stock market prices

The computation of the key parameter values from (Table 2) is shown in (Table 4).

From Table 4, the two stock prices with the smallest/second to the smallest are the 1st and 9th stock prices (SD = 0.15692, Variance = 0.02462) and (SD = 0.41446, Variance = 0.17177) respectively, where 0.5100 and 1.3000 are the carrying capacity of the first and second best stock prices.

Table 3: Computation of the key parameter values of the Model

2011-2016	Carrying Capacity	r (a)	Range	Mean	Variance	SD	b
JAN	34657.2	0.121863	13781.4	27795.86667	25625329	5062.146631	3.52E-06
FEB	39558.9	0.058855	19425.4	28909.8	47279874	6876.036219	1.49E-06
MARCH	38748	0.027822	18095.5	29101.5	45114488	6716.731934	7.18E-07
APRIL	38492.1	0.104685	16446.4	30231.9	39822658	6310.519667	2.72E-06
MAY	41474.4	0.06863	19408	31532.63333	56445845	7513.044971	1.65E-06
JUNE	42482.5	0.18486	20882.9	31380.16667	58052118	7619.194076	4.35E-06
JULY	42097.5	0.175553	19036.1	30848.4	58982012	7679.974745	4.17E-06
AUG	41532.3	0.283818	20034.7	30052.16667	57845765	7605.640352	6.83E-06
SEP	41210.1	0.390831	20837.1	30621.66667	55908833	7477.220916	9.48E-06
OCT	37622.7	0.324103	16687.7	29905.95	43170250	6570.407129	8.61E-06
NOV	38920.9	0.266455	18917.5	28818.78333	46305056	6804.781881	6.85E-06
DEC	41329.2	0.044778	14454.6	32370.95	44659907	6682.806809	1.08E-06

Table 4. The computation of the key parameter values

Carrying Capacity	r	Range	Mean	Variance	SD	b
0.51	1.83333333	0.46	0.183	0.02462333	0.15691824	3.59477124
2.25	0.18181818	2.21	0.467	0.49117889	0.70084156	0.08080808
7.85	4.04761905	7.83	2.426	8.51616	2.91824605	0.51562026
2.3	0.18181818	2.27	0.59	0.71528889	0.84574753	0.07905138
4.8	4.83333333	4.78	0.927	2.07522333	1.44056355	1.00694444
3.4	0.12903226	3.37	0.969	1.14836556	1.07161819	0.03795066
1.6	6.72727273	1.58	0.488	0.36668444	0.60554475	4.20454545
6.55	1.66666667	6.54	0.951	4.46054333	2.11199984	0.25445293
1.3	2.75	1.29	0.332	0.17177333	0.41445547	2.11538462
4.25	1	4.23	0.655	1.73667222	1.3178286	0.23529412

On the basis of these estimated parameter values and simplifying assumptions, we now obtain the full mathematical structure as follows:

$$\frac{dN_1(t)}{dt} = 1.8333(a_1 - 3.5948N_1(t) - b_{12}N_2(t))$$

$$\frac{dN_2(t)}{dt} = 2.7500\,(a_2 - 1.4103N_1(t) - b_{22}N_1(t))$$

and $N_1(0) = 5$, $N_2(0) = 10$

We now carry out numerical stimulations using Maple software.

RESULTS AND DISCUSSION

On the monthly data analysis

From the stimulated analysis, the stock market prices really portray the random nature of stocks over time and shows market and economic function (Figure 1).

On the weekly data analysis

From the behaviour of our simulation results, it showed that, there is an interaction and competition over limited resources in the stock market and this is due to the fact that the weekly data on stock prices gave constraint values (Figure 2).

The two growth rates $N_1(t)$, $N_2(t)$ (1.8333, 2.7500) and its carrying capacity (0.5100 and 1.300) with initial conditions of 5 and 10 moved randomly within some periods before the first growth rate $N_1(t)$ begins to have constant values.

This implies that the carrying capacity is exceeded to the maximum number of consumers in the market it can sustain.

In this study, an application of Non Linear First Order Ordinary Differential Equation Model on stock market analysis has been carried out. This was used to study the effects of growth rates and carrying capacity within the trading period from 2011 to 2016 at monthly intervals and from 6/09/2019 to 15/11/2019 at weekly intervals respectively. Figure 1 and 2 shows good agreement.

We also observe that the growth rate is continuous and

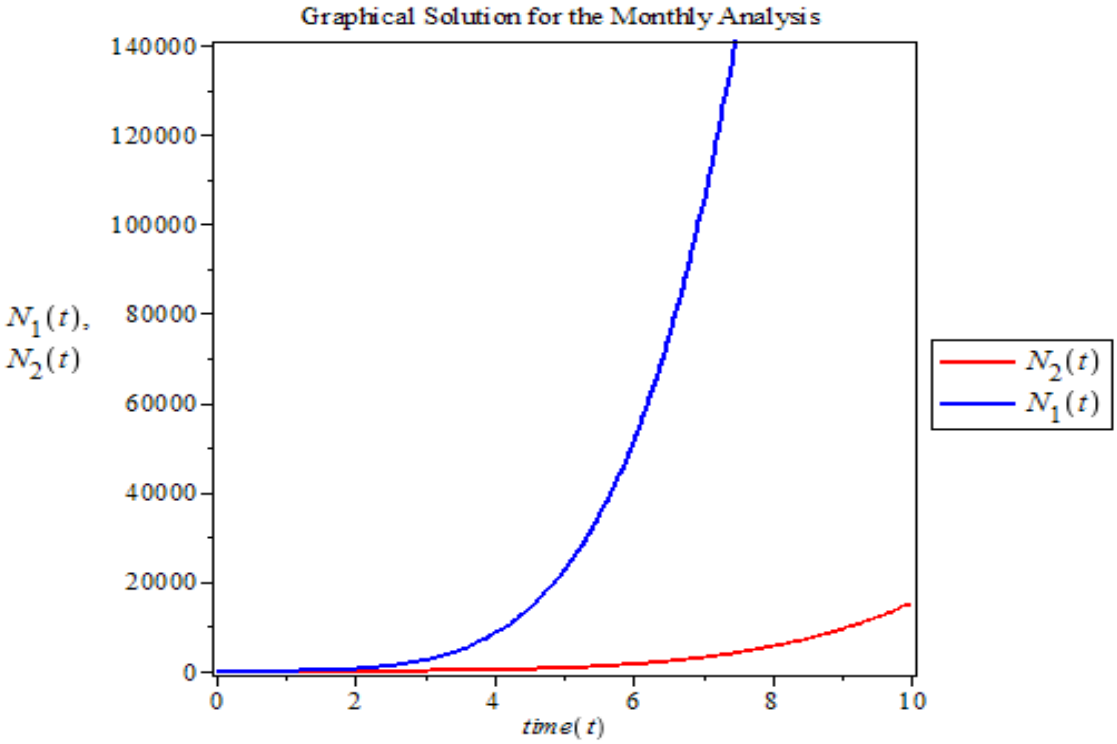

Figure 1: Graphical solution of the monthly analysis.

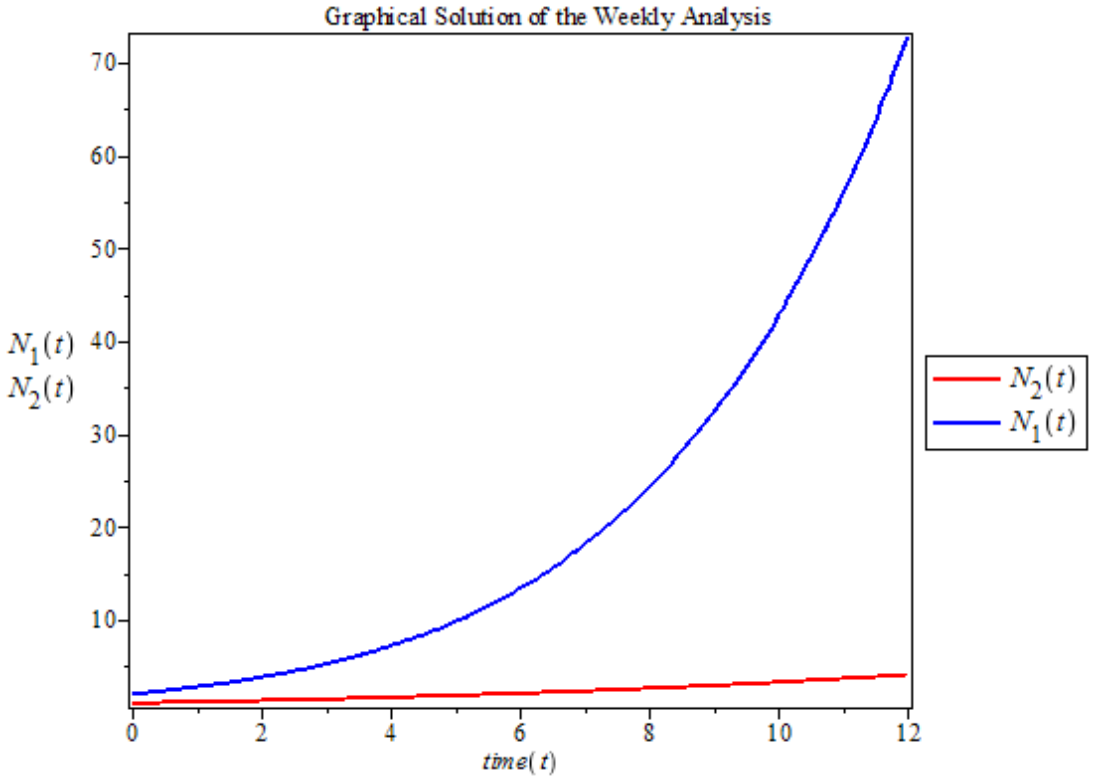

Figure 2: Graphical solution of the weekly analysis.

the competition diverges with increase in time.

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